# X-Ray Analysis of the Structural and Dynamic Properties of $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ Hexagonal Ferrite at Room Temperature 

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#### Abstract

The room temperature crystal structure of $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ hexagonal ferrite has been refined from X-ray single crystal data. This compound is hexagonal, space group $\mathrm{Pb}_{3} / \mathrm{mmc}$, with two formula units per cell and cell parameters $a=5.8920(1) \AA$ and $c=23.183(1) \AA$. The crystal structure has been refined to a final $R$ value of $1.6 \%$ for 380 independent reflections. Three different models are considered for the structural and dynamic characteristics of the bipyramidal Fe ions: (1) a nondisordered configuration, (2) a static disorder between two adjacent pseudotetrahedral sites, and (3) a dynamical disorder between these sites. The X -ray results show that the bipyramidal Fe ions have a disordered configuration and previous Mössbauer spectroscopy studies prove that, at room temperature, the disorder is a dynamical one. The observed oxygen thermal relaxation, Fourier-difference peaks, and interatomic distances are consistent with a fast diffusional motion of the bipyramidal Fe ions within a quasiharmonic double-well potential. © 1985 Academic Press. Inc.


## Introduction

Hexagonal ferrites are a large family of hexagonal or rhombohedral ferrimagnetic oxides with interesting applications as permanent magnets or microwaves devices materials. Their crystal structures can be

[^0]described by the superposition of some fundamental structural blocks formed by a close packing of hexagonal or cubicstacked layers with composition $\mathrm{BaO}_{3}$ and $\mathrm{O}_{4}$. In this framework the metallic ions are located in octahedral and tetrahedral interstices.

The best known of these siructures is the hexagonal M structure of $\mathrm{BaFe}_{12} \mathrm{O}_{19}$, isotype to the mineral magnetoplumbite ( 1 ) which can be described symbolically as

RSR*S* where $R$ is a three-oxygen layer block with composition $\left(\mathrm{Ba}^{2+} \mathrm{Fe}_{6}^{3+} \mathrm{O}_{11}\right)^{2-}$ and S is a two-oxygen layer block with composition $\left(\mathrm{Fe}_{6}^{3+} \mathrm{O}_{8}\right)^{2+}$. The ${ }^{*}$ symbol refers to the fact that the block preceding it has been turned $180^{\circ}$ around the hexagonal $c$ axis by a $6_{3}$ symmetry axis (2). In this crystal structure the iron atoms are distributed within three different kinds of octahedral sites, one tetrahedral site and one trigonal bipyramid site. The hexagonal unit cell of dimensions $a \simeq 5.89 \AA$ and $c \simeq 23.2 \AA$ contains two formulae and is made up of 10 oxygen layers.

The crystal structure of $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ (space group $\mathrm{P6}_{3} / \mathrm{mmc}$ ) was studied for the first time by Townes et al. in 1967 (3). In this work, the fundamental structural characteristics of the magnetopiumbite structure were determined. Nevertheless, some ambiguities remain about the actual structure and dynamic properties of the Fe atoms in the bipyramidal atomic sites. Townes suggested that these cations could be displaced from the $2 d$ position, in the $\mathrm{BaO}_{3}$ mirror plane (model 1), into the $4 e$ positions, 0.156 $\AA$ away from the mirror plane, with half occupation (model 2) (Fig. 1). However, no conclusion was given about the nature of the disorder.

Actually, two possibilities arise for model 2: (a) a statical disorder of two Fe cations within the four tetrahedral equivalent $4 e$ positions (model 2a), or (b) a dynamical disorder with a fast hopping of these cations through the $2 d$ triangular position (model 2b).

The dynamical properties of Fe-bipyramidal atoms and their temperature dependence have been extensively studied by several authors using Mössbauer spectroscopy (4-6). Rensen and van Wieringen (4) and Mamalui et al. (5) have shown that, above $T \simeq 80 \mathrm{~K}$, the bipyramidal ${ }^{57} \mathrm{Fe}$ Lamb-Mössbauer factor is strongly anisotropic, being much smaller in the [001] direction. Below this temperature, the anisot-

a

b

Fig. I. Trigonal bipyramid sublattice: (a) model I and (b) model 2.
ropy of the $f$-factor decreases. Kreber et al. (6) have reported the existence of a discontinuity of the electric quadrupole hyperfine interaction at about the same temperature. The authors interpreted these experimental observations by means of a simple model in which the bipyramidal Fe atom jumps, at high temperature, from one pseudotetrahedral site to the opposite one, and below $T$ $\simeq 80 \mathrm{~K}$ freezes in one of these positions.

A knowledge of the precise structural characteristics of the bipyramidal cations is very important for the theoretical understanding of $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ magnetic anisotropy (7-9). Therefore, we have undertaken a full study of its structural and dynamic properties at low and high temperature. This paper describes an accurate $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ crystal structure refinement at room temperature, based on single crystal X-ray diffraction data and concentrates on three different models for the structural and dynamic properties of the bipyramidal Fe cation.

## Experimental

Single crystals of $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ were grown from a flux melt of composition $\mathrm{NaFeO}_{2}+$ $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ as stated by Gambino and Leonhard (10). A platinum crucible containing the components was placed in a programmable temperature furnace at 1573 K during 24 hr . A controlled cooling rate of $3 \mathrm{~K} / \mathrm{h}$ was then initiated down to 1273 K when the platinum crucible was removed from the furnace and allowed to cool rapidly to room temperature. To extract the crystals the crucible was immersed in a hot
$50 \% \mathrm{HNO}_{3}$ dilute solution. Finally, the crystals were washed in water. In this way, crystals of up to $4 \mathrm{~cm}^{3}$ were obtained. In order to verify that the M phase was unique some X-ray precession photographs were obtained. These precession photographs show all the crystals to be hexagonal with $a$ $\simeq 5.89 \AA$ and $c \simeq 23.2 \AA$. Taking into account the Laue symmetry, the number of atoms per unit cell and the systematic extinctions observed, $h h l, l=2 n+1$, only $\mathrm{P} \overline{6} 2 c, P 6_{3} m c$, and $P 6_{3} / m m c$ space groups are possible.

For the intensity data collection, a sphere of radius 0.104 mm was mounted on a Philips PW1100 4-circle X-ray diffractometer, using $\mathrm{Ag} K \alpha$ radiation. Least-squares refinement of twelve $2 \theta$ reflection values, in the range $19^{\circ}<\theta<37^{\circ}$ where complete $\alpha_{1} /$ $\alpha_{2}$ separation occurs, gave the lattice parameters $a=5.8920(3) \AA$ and $c=23.183(1)$ $\AA$, in good agreement with those determined by Townes (3). All reflections from a hemisphere in the interval of $4-28^{\circ}$ were measured by the $\omega$-scan technique with a variable scan width $\Delta \theta=(1.40+0.20 \tan \theta)$ at the speed of $0.20^{\circ} / \mathrm{sec}$. The background was measured at each end of the interval for a time $t=\left(T_{\mathrm{sc}} / 2\right)\left(I_{\mathrm{bg}} / I_{\mathrm{nt}}\right)^{1 / 2}$, where $T_{\mathrm{sc}}=$ scan time, $I_{\text {bg }}=$ background intensity, $I_{\mathrm{nt}}=$ net intensity. A total of 4517 reflections were measured which gave, after averaging according to the $6 / \mathrm{mmm}$ Laue class, 631 independent reflections. The intensities were converted to structure factors by applying Lorentz polarization and absorption ( $\mu R=0.818$ ) corrections. Finally, 447 reflections with $\left|F_{\text {obs }}\right|>5 \sigma\left(F_{\text {obs }}\right)$ were retained. Scattering factor curves for neutral $\mathrm{Ba}, \mathrm{Fe}$, and O given by Doyle and Turner (11) and anomalous dispersion data of Cromer and Liberman (12) were used in the crystal structure refinement.

The structural refinement was carried out with the Enraf-Nonius structure determination package (13) and the LINEX leastsquares program (adapted from ORFLS)
(14). An isotropic secondary extinction correction type I (15) with a Lorentzian or Gaussian mosaic distribution law has been applied with the LINEX program. The obtained extinction factor value was $G=$ $0.022(2)$, which corresponds to a mean angular dispersion of the mosaic of 145 sec . Similar final positional and thermal parameters were obtained with the Enraf-Nonius programs taking into consideration reflections such as $\sin \theta / \lambda>0.35$ and rejecting the 15 strongest reflections, 380 reflections remained in this way. The weights attributed to the reflections during the structure refinement were $w=1 / \sigma^{2}\left(F_{0}\right)$ where

$$
\begin{aligned}
\sigma\left(F_{0}\right)=\sigma\left(F_{0}^{2}\right) / 2 & F_{0}, \sigma\left(F_{0}^{2}\right) \\
& =\left[\sigma^{2}\left(I_{0}\right)+\left(0.03 I_{0}^{2}\right)\right]^{1 / 2} / L P
\end{aligned}
$$

where $\sigma\left(I_{0}\right)$ is the counting statistics standard deviation.

The structure refinements performed in the noncentrosymmetric $P 6_{3} m c$ and $P \overline{6} 2 c$ space groups have given nonconclusive results; therefore, the centrosymmetric $P 6_{3} / m m c$ space group has been adopted. The atomic positions reported by Wyckoff (16) have been taken.

Two different refinements have been performed, one with the bipyramidal Fe atoms in the $2 d$ positions (model 1), and another with these atoms in the $4 e$ positions with half occupancy (model 2). In Table I, we report the results obtained for both models with anisotropic and isotropic temperature factors. Atomic position coordinates and temperature factors for the atoms other than $\mathrm{Fe} 2, \mathrm{O} 1$, and O 3 do not change, within their standard deviations, with the choice of the model. The obtained $R$-factors for each case are also indicated in Table I. We note that the best $R$-factors are those of the model 2 with anisotropic temperature factors ( $R=0.016, R_{\mathrm{w}}=0.021$ ).

## Discussion

The interatomic distances and angles

TABLE I
Position and Thermal Isotropic and Anisotropic ( $\times 10^{5}$ ) Parameters

|  | Position | $x$ | $y$ | $z$ | $\beta_{11}$ | $\beta_{22}$ | $\beta_{33}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{23}$ | $\begin{gathered} B \\ \left(\mathrm{~A}^{2}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ba | $2 d$ | 2/3 | 1/3 | 0.25 | 556(7) | 556 | 26(1) | 278 | 0 | 0 | 0.57(1) |
| Fel | $2 a$ | 0 | 0 | 0 | 480(18) | 480 | 13(1) | 240 | 0 | 0 | 0.41 (1) |
| Fe 2 | $4 e^{a}$ | 0 | 0 | 0.25733(4) | 420(20) | 420 | 31(2) | 210 | 0 | 0 | 0.43 (1) |
|  | $2 b^{\text {b }}$ | 0 | 0 | 0.25 | 365(24) | 365 | 172(3) | 182 | 0 | 0 | 1.18(3) |
| Fe3 | $4 f$ | 1/3 | 2/3 | 0.02713(2) | 417(2) | 417 | 19(1) | 208 | 0 | 0 | 0.42(1) |
| Fe4 | $4 f$ | 1/3 | 2/3 | 0.19030(2) | 473(1) | 473 | 19(1) | 236 | 0 | 0 | 0.46(1) |
| Fe5 | $12 k$ | 0.16868 | $0.33735(7)$ | -0.10825(1) | 455(6) | 395(2) | 24(1) | 197 | 2 | $5(2)$ | 0.47 (1) |
| 01 | $4 e^{a}$ | 0 | 0 | $0.15094(13)$ | 472(67) | 472 | $40(4)$ | 236 | 0 | 0 | $\begin{aligned} & 0.61(5) \\ & 0.47(7) \end{aligned}$ |
| O2 | $4 f$ | 1/3 | 2/3 | -0.05454(12) | 622(67) | 622 | 13(3) | 311 | 0 | 0 | 0.48(5) |
| O3 | $\begin{aligned} & 6 h^{a} \\ & 6 h^{b} \end{aligned}$ | 0.18213 | 0.36426(47) | 0.25 | 778(50) | 389(58) | $\begin{aligned} & 24(3) \\ & 30(4) \end{aligned}$ | 194 | 0 | 0 | $0.66(5)$ $0.82(7)$ |
| 04 | $12 k$ | 0.15647 | 0.31294(35) | $0.05192(8)$ | 567(3) | 487(40) | 24(2) | 243 | 3 | 7(9) | $0.56(3)$ |
| O5 | $12 k$ | 0.50260 | 1.00520(34) | $0.14957(8)$ | 614(32) | 638(38) | 30(2) | 319 | 23 | 46(9) | 0.68(3) |

Note. $T=\exp \left[-\sum_{i j} \beta_{i j} h_{i} h_{j}\right]=\exp -B(\sin \theta / \lambda)^{2}$, by symmetry $\beta_{11}=\beta_{22}=2 \beta_{12}$ and $\beta_{13}=\beta_{23}=0$ for $2 d$,
$2 a, 4 e, 4 f$ positions; $\beta_{22}=2 \beta_{12}$ and $\beta_{23}=2 \beta_{13}$ for $12 k$ position; $\beta_{22}=2 \beta_{12}$ and $\beta_{13}=\beta_{23}=0$ for $6 h$ position.
${ }^{a}$ Model 2: $R=0.016, R_{\mathrm{w}}=0.021$ for anisotropic temperature factors. $R=0.022, R_{\mathrm{w}}=0.032$ for isotropic temperature factors.
${ }^{b}$ Model 1: $R=0.018, R_{\mathrm{w}}=0.025$ for anisotropic temperature factors. $R=0.042, R_{\mathrm{w}}=0.054$ for isotropic temperature factors.

TABLE II
Interatomic Distances $(\AA)$ and Angles $\left({ }^{\circ}\right)$ in $\mathrm{BaF}_{12} \mathrm{O}_{19}$

| Ba polyhedron |  |  | Fe 2 bipyramid |  |
| :---: | :---: | :---: | :---: | :---: |
| Ba-O3 | $\times$ | $6=2.950(2)$ | Fe 2 (I)-O1(IV) | $\times 1=2.128(3)(2.298(3))^{b}$ |
| $\mathrm{Ba}-05$ | $\times$ | $6=2.868(2)$ | $\mathrm{Fe} 2(\mathrm{I})-\mathrm{Ol}$ (I) | $\times 1=2.468(3)(2.298(3))^{b}$ |
| (Ba-O) |  | 2.909 | $\mathrm{Fe} 2-\mathrm{O} 3$ | $\times 3=1.867(2)(1.859(2))^{b}$ |
| O3(I)-O3(II) | $\times$ | $3=2.672(3)$ | ( $\mathrm{Fe} 2-\mathrm{O}$ ) | $2.039(2.035)^{b}$ |
| O3(I)-O3(III) | $\times$ | $3=3.220(3)$ | Fe2(I)-Fe2(IV) | $\times 1=0.340(1)$ |
| O5(I)-03(II) | $\times$ | 6 $=2.900(2)$ | O1-O3 | $\times 6=2.955(3)$ |
| O5(I)-O5(III) |  | $12=2.850(3)$ | O3(I)-O3(II) ${ }^{\text {c }}$ | $\times 3=3.220(3)$ |
| O3(I) -Ba(I)-O3(II) | $\times$ | $3=60.1(1)$ | $\mathrm{O} 1-\mathrm{O} 3-\mathrm{Ol}$ | $\times 3=102.1(1)$ |
| O5(III)-03(I)-O5(XII) | $\times$ | $6=109.6(1)$ | Fe 3 tetrahedron |  |
| O3(II) $-\mathrm{O5}$ (I) - O 3 (III) | $\times$ | $6=55.9(1)$ | $\mathrm{Fe} 3-\mathrm{O} 2$ | $\times 1=1.894(3)$ |
| O3(I) - $\mathrm{Ba}(\mathrm{I})-\mathrm{O} 5(\mathrm{III})$ |  | $12=58.6(1)$ | $\mathrm{Fe} 3-\mathrm{O} 4$ | $\times 3=1.894(2)$ |
| Fel octahedron |  |  | O4(I)-O4(II) | $\times 3=3.126(2)$ |
| Fe1-O4 | $\times$ | $6=2.000(2)$ | O4(1)-O2(l) | $\times 3=3.058(3)$ |
| O4(VII)-O4(VIII) ${ }^{\text {a }}$ | $\times$ | $6=2.766(2)$ | O2-Fe3-O4 | $\times 3=107.7(1)$ |
| O4(VII)-O4(XI) | $\times$ | $6=2.889(3)$ | O4-O2-O4 | $\times 3=61.5(1)$ |
| O4(IX)-Fe1(II)-04(XII) | $\times$ | $6=87.5(1)$ | Fe4 octahedron |  |
| O4(IX)-Fe1(II)-O4(VIII) | $\times$ | $6=92.5(1)$ | Fe4-O3 | $\times 3=2.073(2)$ |

TABLE II-Continued

| Fe4-O5 | $\times$ | $3=1.969(2)$ | Important distances and angles |  |
| :---: | :---: | :---: | :---: | :---: |
| (Fe4-O) |  | 2.021 | $\mathrm{Ba}(\mathrm{I})$-Fe4(I) | $=3.673(1)$ |
| O3(I)-O3(II) ${ }^{\text {d }}$ | $\times$ | $3=2.672(3)$ | $\mathrm{Ba}(\mathrm{I})-\mathrm{Fe} 2$ (I) | $=3.406(1)$ |
| O3(I)-05(III) ${ }^{e}$ | $\times$ | $6=2.850(3)$ | $\mathrm{Ba}(\mathrm{II})-\mathrm{Fe} 5$ (I) | $=3.691(1)$ |
| O5(I)-O5(III) | $\times$ | $3=2.992(3)$ | Fel(I) -Fe3(I) | $=3.460(1)$ |
|  |  |  | $\mathrm{Fe} 1(\mathrm{I})-\mathrm{Fe} 5(\mathrm{I})^{i}$ | $=3.044(1)$ |
| O3(I)-Fe4(I)-O5(I) | $\times$ | $3=166.8(\mathrm{I})$ | $\mathrm{Fe} 2(\mathrm{I})-\mathrm{Fe} 4$ (I) | $=3.740(1)$ |
| O3(I)-Fe4(I)-O5(III) | $\times$ | $6=89.6(1)$ | $\mathrm{Fe} 2(\mathrm{II})-\mathrm{Fe} 5(\mathrm{III})$ | $=3.862(1)$ |
| O3(I)-Fe4(I)-O3(II) | $\times$ | $3=80.3(1)$ | Fe 2 (III)-Fe5(III) | $=3.560(1)$ |
| O5(I)-Fe4(I)-O5(III) | $\times$ | $3=98.9(1)$ | $\mathrm{Fe} 3(\mathrm{I})-\mathrm{Fe} 5(\mathrm{I})$ | $=3.561(1)$ |
|  |  |  | Fe 3 (II) - Fe 5 (I) | $=3.495(1)$ |
| Fe5 octahedron |  |  | Fe4(I) -Fe4(IV) ${ }^{\text {j }}$ | $=2.768(1)$ |
| Fe5-OI | $\times$ | $1=1.985(2)$ | Fe5(I) - $\mathrm{Fe} 5(\mathrm{II})^{i}$ | $=2.911(1)$ |
| Fe5-O2 | $\times$ | $1=2.092(2)$ | $\mathrm{Fe} 5(\mathrm{I})-\mathrm{Fe} 5(\mathrm{III})^{i}$ | $=2.982(1)$ |
| Fe5-O4 | $\times$ | $2=2.114(3)$ | $\mathrm{Fe} 2(\mathrm{I})-\mathrm{Fe} 4(\mathrm{IV})$ | $=3.612(1)$ |
| Fe5-O5 | $\times$ | $2=1.932(3)$ | Fel(II) -O4(XI)-Fe3(III) | $=126.3(1)$ |
| (Fe5-O) |  | 2.028 | $\mathrm{Fe} 1(\mathrm{I})-\mathrm{O} 4(\mathrm{~V})-\mathrm{Fe} 5(\mathrm{I})$ | $=95.4(1)$ |
| O2(I) - 04 (V) | $\times$ | $2=2.949(3)$ | Fe2(III)-O1(II) -Fe5(III) | $=119.9$ (1) |
| O2(I) -05(V) ${ }^{\text {r }}$ | $\times$ | $2=2.768(3)$ | Fe 2 (I) -O3(III)-Fe4(IV) | $=132.9(1)$ |
| Ol(II) - $04(\mathrm{~V})^{r}$ | $\times$ | $2=2.796(3)$ | $\mathrm{Fc} 2(\mathrm{I})-\mathrm{O} 3(\mathrm{LII})-\mathrm{Fe} 4(\mathrm{I})$ | $=143.3$ (1) |
| O1(II)-O5(V) | $\times$ | $2=2.946(3)$ | $\mathrm{Fe} 3(\mathrm{I})-\mathrm{O} 2(\mathrm{I})-\mathrm{Fe} 5(\mathrm{I})$ | $=126.5(1)$ |
| O4(V)-O4(VI) ${ }^{\text {R }}$ | $\times$ | $1=2.766(3)$ | Fe3(II) -O4(V) -Fe5(I) | $=121.3(1)$ |
| O4(V)-O5(VI) | $\times$ | $2=2.873(3)$ | Fe4(I) -O3(I) -Fc4(IV) | $=83.8(1)$ |
| $\mathrm{O}(\mathrm{V})-\mathrm{O} 5(\mathrm{VI})^{h}$ | $\times$ | $1=2.900(3)$ | Fe4(II) -O5(VI)-Fe5(I) | $=128.1(1)$ |
|  |  |  | Fe5(I) -OI(II) -Fe5(III) | $=97.3(1)$ |
| O2(I) -Fc5(I)-O1(II) | $\times$ | $1=173.3(1)$ | $\mathrm{Fe} 5(\mathrm{I})-\mathrm{O} 2(\mathrm{I})-\mathrm{Fe} 5(\mathrm{III})$ | $=88.2(1)$ |
| O2(I) -Fe5(I)-O4(V) | $\times$ | $2=89.0(1)$ | $\mathrm{Fe} 5(\mathrm{I})-\mathrm{O} 5(\mathrm{~V})-\mathrm{Fe} 5(\mathrm{III})$ | $=97.8(1)$ |
| O2(I) -Fe5(I)-O5(V) | $\times$ | $2=86.8(1)$ | $\mathrm{Fe} 5(\mathrm{I})-\mathrm{O} 4(\mathrm{~V})-\mathrm{Fe} 5(\mathrm{III})$ | $=89.7(1)$ |
| O1(II)-Fe5(I)-O4(V) | $\times$ | $2-85.9(1)$ |  |  |
| O1(II)--Fe5(I)-O5(V) | $\times$ | $2=97.6(1)$ |  |  |

${ }^{\text {a }}$ Shared edge with Fe 5 octahedra.
${ }^{b}$ Model 1.
c Shared edge with Ba polyhedron.
${ }^{d}$ Shared face with Fe4 octahedra and edge with Ba polyhedron.
${ }^{e}$ Shared face with Ba polyhedron.
${ }^{f}$ Shared edge with Fe 5 octahedron.
${ }^{s}$ Shared edge with Fel octahedron.
${ }^{h}$ Shared edge with Ba polyhedron.
${ }^{i}$ Octahedra with a shared edge.
Octahedra with a shared face.
with their standard deviations for models 1 and 2 are given in Table II. These values were calculated by the BONDLA program of the X-RAY system (17). Roman numbers correspond to the equivalent position apparition order in the International Tables for X-Ray Cristallography (18). In Table III we report the thermal parameters obtained for models 1 and 2 . The coordination polyhedra and the way they link together are
shown in Fig. 2. In order to know the way in which the local electric neutrality is accomplished, we have calculated the effective valency of the different ions throughout the structure, taking into account the empirical bond-length to bond-strength relation proposed by Brown and Kun Wu (19): $S_{\mathrm{ij}}=\left(D_{i j} / D_{1}\right)^{-N}$ where $D_{1}$ and $N$ are constants depending on the different cations, $S_{i j}$ is the bond strength and $D_{i j}$ is the inter-

TABLE III
Thermal Ellipsoid Data

| Atom | Symmetry | rms (Å) | Angles ( ${ }^{\circ}$ ) with the hexagonal axes |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $a$ | $b$ | c |
| Ba | $\overline{6} m 2$ | 0.09 |  | opic |  |
| Fe 1 | 3 m | $0.08{ }^{\text {a }}$ | 0 | 0 | 90 |
|  |  | 0.06 | 90 | 90 | 0 |
| Fe 2 | $3 m,{ }^{\text {b }} \mathbf{6} m 2^{\text {c }}$ | $0.09,{ }^{\text {b }} 0.22^{\text {c }}$ | 90 | 90 | 0 |
|  |  | $0.08,{ }^{\text {a }} 0.07{ }^{a}$ | 0 | 0 | 90 |
| Fe3 | $3 m$ | 0.07 | Iso | opic |  |
| Fe4 | $3 m$ | $0.08{ }^{\text {a }}$ | 0 | 0 | 90 |
|  |  | 0.07 | 90 | 90 | 0 |
| Fe5 | $m$ | 0.08 | 90 | 77 | 15 |
|  |  | 0.08 | 0 | 120 | 90 |
|  |  | 0.07 | 90 | 33 | 105 |
| 01 | $3 m$ | 0.10 | 90 | 90 | 0 |
|  |  | $0.08{ }^{\text {a }}$ | 0 | 0 | 90 |
| O2 | $3 m$ | $0.09{ }^{\text {a }}$ | 0 | 0 | 90 |
|  |  | 0.06 | 90 | 90 | 0 |
| O3 | $m m$ | 0.11 | 0 | 120 | 90 |
|  |  | 0.08 | 90 | 90 | 0 |
|  |  | 0.07 | 90 | 30 | 90 |
| 04 | $m$ | 0.09 | 0 | 120 | 90 |
|  |  | 0.09 | 90 | 55 | 41 |
|  |  | 0.08 | 90 | 49 | 131 |
| O5 | $m$ | 0.11 | 90 | 51 | 47 |
|  |  | 0.09 | 0 | 120 | 90 |
|  |  | 0.07 | 90 | 53 | 137 |

${ }^{a}$ Circular section $\perp_{2}$ due to symmetry.
${ }^{b}$ Model 2.
${ }^{c}$ Model 1.
atomic distance. In this way, the effective valencies for the cations and anions are, respectively, $V_{\mathrm{c}}=\sum_{j=1}^{n} S_{i j}$ and $V_{\mathrm{a}}=\sum_{i=1}^{m} S_{i j}$. The results of these calculations for model 2 , with the standard deviations, are reported in Table IV.

## R-Block Polyhedra

The R-block, with mirror symmetry at $z$ $=1 / 4$, contains two oxygen layers surrounding the $\mathrm{BaO}_{3}$ layer, and the Fe 2 and Fe 4 cations (Fig. 3). The Fe 5 octahedra,
shared with the neighboring S-blocks, are the limits of the block.

The Ba site has 12 -fold coordination with two sets of $\mathrm{Ba}-\mathrm{O}$ distances: six longer bonds forming an irregular hexagon in the mirror plane ( $d=2.950 \AA$ ) and six shorter distances with the oxygen in the neighboring layers ( $d=2.868 \AA$ ), leading to a slightly axially compressed polyhedron. The mean $\mathrm{Ba}-\mathrm{O}$ distance $(d=2.909 \AA)$ is somewhat shorter than what could be expected from $\mathrm{Ba}^{2+}$ effective ionic radius $r^{\mathrm{xII}}$ $\left(\mathrm{Ba}^{2+}\right)=1.60 \AA(20)$, consequently the calculated valency is about $16 \%$ greater than the theoretical one. Thermal vibration of Ba cation is normal and isotropic.
Beside Ba polyhedron, Fe 4 cations form a $\left(\mathrm{Fe}_{2} \mathrm{O}_{9}\right)$ group of two octahedra sharing a face in the mirror plane at $z=1 / 4$. In addition, each octahedron shares three faces with different Ba polyhedra. In this configuration the $\mathrm{Fe}^{3+}-\mathrm{Fe}^{3+}$ electrostatic repulsion separates the cations from each other and the octahedron becomes distorted, with three O3-O3 shorter distances (shared


Fig. 2. Coordination polyhedra of cations in Ba $\mathrm{Fe}_{12} \mathrm{O}_{19}$.

TABLE IV
Bond Strengths and Effective Ionic Valencies

| $S_{i j}=S_{j i}$ | O1 | O2 | O3 | O4 | O5 | Effective valency |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ba |  |  | 0.174( $\times 6$ ) |  | $0.211(\times 6)$ | 2.31(1) |
|  |  |  | $(\times 2)$ |  | ( $\times 1$ ) |  |
| Fel |  |  |  | 0.515( $\times 6$ ) |  | 3.09(2) |
|  |  |  |  | $(\times 1)$ |  |  |
| Fe 2 | 0.362( $\times 1$ ) |  | 0.762( $\times 3$ ) |  |  | 2.80(2) |
|  | $(\times 1 / 2)$ |  | $(\times 1)$ |  |  |  |
|  | $0.155(\times 1)$ |  |  |  |  |  |
|  | $(\times 1 / 2)$ |  |  |  |  |  |
| Fe3 |  | 0.702( $\times 1$ ) |  | 0.702( $\times 3$ ) |  | 2.81(2) |
|  |  | $(\times 1)$ |  | $(\times 1)$ |  |  |
| Fe4 |  |  | $0.420(\times 3)$ |  | $0.563(\times 3)$ | 2.95(2) |
|  |  |  | $(\times 2)$ |  | ( $\times 1$ ) |  |
| Fe5 | 0.537(×1) | 0.398( $\times 1$ ) |  | $0.375(\times 2)$ | $0.627(\times 2)$ | 2.94(2) |
|  | $(\times 3)$ | $(\times 3)$ |  | $(\times 2)$ | $(\times 2)$ |  |
| Effective valency | 1.87 (1) | 1.90 (2) | 1.95(1) | $1.97(2)$ | 2.03(2) |  |
|  |  |  |  |  |  |  |

face, $d=2.672 \AA$ ) and three $\mathrm{O} 5-\mathrm{O} 5$ longer distances ( $d=2.992 \AA$ ).

Finally, the R-block contains the Fe 2 trigonal bipyramid, a not very common coordination in ferric oxides. This trigonal bipyramid can be regarded as two tetrahedra sharing a face in the mirror plane. These tetrahedra are axially compressed with O3O 3 distances $(~ d=3.220 \AA$ ) much larger than the O3-O1 ones ( $d=2.953 \AA$ ). These enhanced $\mathrm{O} 3-\mathrm{O} 3$ distances match the correspondingly reduced $\mathrm{O} 3-\mathrm{O} 3$ distances in


Fig. 3. R-Block polyhedra projected in the $Z=0.25$ mirror plane.
the shared face of the neighboring Fe 4 octahedra, leading to the irregular hexagon surrounding Ba cation in the $z=1 / 4$ mirror plane.

It is our purpose to discuss in this paper the consistency of our X-ray data and of previous Mössbauer spectroscopy results (4-6) with the three different possibilities for the location and the dynamic characteristics of the bipyramidal Fe 2 cations. The X-ray data can help us in the discussion of this problem by means of the study of (i) $R$ factors and residuals in the Fourier-difference synthesis, (ii) thermal relaxation of surrounding oxygen, and (iii) interatomic distances.

The important difference between $R$-factors obtained by least-squares refinements of model $1\left(R_{\mathrm{w}}=0.025\right)$ and model $2\left(R_{\mathrm{w}}=\right.$ 0.021 ) gives a high level of statistical significance to the new $z$ parameter introduced for the Fe 2 atoms as shown by the Hamilton test (21). This result enables us to rule out model 1: Fe 2 atom has not a rms thermal amplitude of $0.22 \AA$ along $c$-axis within a single harmonic potential. This conclu-
sion is confirmed by the appearance, around Fe 2 position, of several strong peaks in the Fourier-difference synthesis performed after the least-squares refincment of model 1 (Table V).

At this point, it is clear that only the disordered configurations corresponding to model 2 must be considered. As has been said before, two different possibilities exist for the disorder characteristics of Fe 2 atom: a static disorder (model 2a) and a dynamic disorder (model 2b).

It is well known that experiments on elastic scattering of radiation (photons or neutrons) do not furnish any information about the temporal behavior of the atoms: the same atomic positions and thermal parameters would be obtained for a single atom presenting a dynamic or a static disorder, after a crystal structure refinement based on the diffracted Bragg intensities. Nevertheless, we must bear in mind that the consequences of the atomic disorder on its environment may really differ in the two cases. The Mössbauer effect, instead, can give valuable information about the temporal behavior of the atoms. The phase incoherence of photons emitted or absorbed by diffusing Mössbauer nuclei produce a

TABLE V
Fourier-Difference Peaks around Fe2 Atom after Model 1 and Model 2 Refinements

| Model $1^{\text {a }}$ |  | Model $2^{\text {b }}$ |  |
| :---: | :---: | :---: | :---: |
| $Z^{\text {c }}$ $(\AA)$ | Electronic density ( $e-/ \AA^{3}$ ) | $\begin{gathered} Z \\ (\AA) \end{gathered}$ | Electronic density (e-/ $/ \AA^{3}$ ) |
| 2.71 | +0.48 | 2.71 | +0.42 |
| 2.08 | -0.78 | 2.00 | -0.59 |
| 1.04 | +1.07 | 0.00 | +0.28 |
| 0.70 | -1.92 |  |  |
| 0.35 | +0.93 |  |  |

[^1]broadened quasielastic resonant component in the Mössbauer spectra $(22,23)$ from which a great variety of dynamic behaviors can be studied (24-27). When the inverse of the atom jumping frequency is comparable to the characteristic Mössbauer time ( $\tau$ $\sim 10^{-8} \mathrm{sec}$ for ${ }^{57} \mathrm{Fe}$ ), the quasielastic component is made evident in the Mössbauer spectra and when the jumping frequency is greater than the inverse of the Mössbauer time, the quasielastic component is smeared out in the background resulting only in a reduced effective recoil-free fraction. Therefore, we interpret the low $f$-factors of the bipyramidal ${ }^{57} \mathrm{Fe}$ nuclei observed by Rensen and van Wieringen (4) and Mamalui et al. (5), at high temperatures, as being due to a dynamical disorder configuration (model 2 b ). The nonobservance of a line broadening in the Mössbauer spectra obtained by Mamalui et al. (5) and Kreber et al. (6) at temperatures close to $T=80 \mathrm{~K}$ may be due to the strong overlap of the bipyramidal subspectra with the remaining sublattices subspectra.

Recapitulating, we can say that only the consideration of both X-ray and Mössbauer spectroscopy results, enables us to conclude that model 1 and model 2 a must be disregarded: X-ray results allow us to distinguish between model 1 and model 2 , and Mössbauer spectroscopy allows us to disregard model 2 a .

Now we will discuss the consistency of the remaining model $2 b$ with the observed thermal, Fourier-map, and structural features.

Thermal ellipsoid parameters of atoms belonging to Fe 2 and Ba polyhedra can be observed in Fig. 4 and Table III. We note that O 1 ion has its largest vibration along the hexagonal $c$-axis (rms $0.11 \AA$ ) whereas O 3 anion has its strongest vibration along the $a$-axis (rms $0.11 \AA$ ), perpendicular to the mirror plane containing the $\mathrm{Fe} 2-\mathrm{O} 3$ and the $\mathrm{Fe} 4-\mathrm{O} 3$ strong bonds. The fact that O1 ions have a large thermal motion along the


Fig. 4. Thermal ellipsoids in Ba and Fe 2 polyhedra (model 2).
trigonal axis of the bipyramid is consistent with the diffusion motion of Fe 2 atom along this axis. We remark that the same result would be obtained if model 1 (thermal vibration) or model 2a (mean static displacement) were right. With regard to O 3 atoms, their thermal characteristics must be discussed together with the observed interatomic distances. A close relationship is expected to exist between equilibrium position and thermal parameters of O 3 ion and the jumping frequency of the neighboring Fe 2 atoms. If the jumping frequency were low, the O3 equilibrium positions would be near those expected for a static disorder configuration (model 2a) and if the jumping frequency were high the equilibrium positions would be those expected for the model 1 configuration ( Fe 2 centered in the bipyramid). In the first case, a large thermal relaxation would exist, while in the second case it would be smaller.

As we have said above, the two unequivalent $\mathrm{O} 3-\mathrm{O} 3$ distances in the $\mathrm{BaO}_{3}$ mirror plane are very different (3.220 and $2.672 \AA$ ). Additional evidence of the fact that these O3-O3 distances are very unusual can be obtained from their comparison with those found in other structures with cquivalent local configurations. For example, in the T-block of the hexagonal ferrites (2) an equivalent structural configuration does exist, with the unique difference of the bipyramidal site which has been converted to a tetrahedral one. The observed oxygen distances are in this case (28): $d(\mathrm{O} 3-\mathrm{O} 3)=$
$2.737 \AA$ ( $2.672 \AA$ in R-block) for the common face of the $\mathrm{Fe}_{2} \mathrm{O}_{9}$ group and $d(\mathrm{O} 3-\mathrm{O} 3)$ $=3.139 \AA$ ( $3.220 \AA$ in R-block) for the base of the tetrahedron (bipyramid). Therefore, it can be concluded that some additional distorting factors exist in the R -block of the $\mathrm{BaFe}_{12} \mathrm{O}_{19}$ structure. In this block the O 3 atoms of the bipyramid base must move away in order to make the minimum opening of the Fe 2 passageway ( $1.895 \AA$ ) compatible with the $\mathrm{Fe}^{3+}$ ionic radius. For example, the $\mathrm{Fe}^{3+}-\mathrm{O}$ distance in YIG tetrahedral sites is $1.865 \AA$ (29). When the O3 atoms are located at these new equilibrium positions they vibrate along the weaker bond with a normal amplitude (see Table III).

The validity of the adopted model can also be checked by performing a Fourierdifference synthesis after the crystal structure refinement. By this procedure we found that only around the bipyramidal position did some peaks appear (see Table V). From the comparison of the peaks appearing after the refinement of model 1 and model 2 it is seen that, effectively, model 2 gives a better agreement with the observed electron density. Two peaks, of opposite sign, exist at each side of O 1 ion which remain, independently of the chosen model. If we take into account that Ol ion is not placed in a symmetry center, we may interpret these peaks as arising from anharmonicity effects (odd powers of the atomic potential). In this way, the minus sign appearing on the $\mathrm{Fe} 2-\mathrm{O} 1$ bond indicates that a steepest potential does exist at this side.

All these structural and thermal features are consistent with a dynamical disorder of the Fe 2 atom (model 2b). Furthermore, they indicate that Fe 2 atom oscillates very fast in the double-well potential. As a last remark we suggest that more insight on this disordered configuration could be obtained from a study of the temperature dependence of their structural and thermal fea-
tures as deduced from temperature variable X-ray diffraction data (30).

## S-Block Polyhedra

The spinel block consists of two oxygen layers stacked in a cubic close-packing fashion with the neighboring layers belonging to the R-block. These two inner layers contain 2 tetrahedral sites ( Fe 3 ) and one octahedral site ( Fe 1 ) while in the limits of the block 3 octahedral sites ( Fe 5 ) are found (Fig. 2). The stacking of this $S$-block along $c$-axis is the same as the well-known cubic spinel structure along [111] cubic axes. The cationic octahedra share edges among themselves and corners with tetrahedra.

The interatomic distances in the M-structure $S$-block polyhedra are roughly the same as those in the spinel structure (Table II), except for Fe 5 octahedra which are more distorted with point symmetry $m$. The thermal vibrations of cations and anions in this block are normal having orientations such that the strongest bonds are avoided. Fel cation has a greater thermal motion within the (001) plane because the interlayer $\mathrm{O} 4-\mathrm{O} 4$ distance is smaller ( $d=2.766$ $\AA$ ) than the $04-04$ distance in the layer itself ( $d=2.889 \AA$ ). The tetrahedral Fe 3 cation vibrates isotropically because of its isotropic bond strengths. The strength of the $\mathrm{Fe} 3-\mathrm{O} 2$ bond along $z$ gives to O 2 a larger thermal motion in the (001) plane than along the $c$-axis.

## Conclusions

Our X-ray single crystal study of $\mathrm{BaFe}_{12}$ $\mathrm{O}_{19}$ has defined more precisely the structural characteristics proposed by Townes et al. (3). The dynamical characteristics of the bipyramidal Fe ions have been extensively discussed. From the consideration of our own X-ray results and of the preexisting Mössbauer effect results, the distinction between three a priori structural models have been accomplished. X-Ray diffraction
study have enabled us to rule out model 1, in which Fe ions would be found in the center of the trigonal bipyramid with a high thermal motion along the trigonal axis within a single harmonic potential. The existing Mössbauer spectroscopy results allow us to disregard the static disorder configuration (model 2a). In this way, we conclude that the bipyramidal Fe ions stay in a double-well quasiharmonic atomic potential with the two minima at each side of the $\mathrm{BaO}_{3}$ mirror plane, $0.17 \AA$ away. Within this effective atomic potential the Fe ion presents, at room temperature, a fast diffusional motion between two pseudotetrahedral sites, inducing a supplementary enlargement of the bipyramid base.

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[^1]:    ${ }^{a} R=0.018 ; R_{\mathrm{w}}=0.025$.
    ${ }^{b} R=0.016 ; R_{\mathrm{w}}=0.021$.
    ${ }^{c}$ Distance from the $\mathrm{BaO}_{3}$ mirror plane; $\boldsymbol{Z}(\mathrm{OI})=$ 2.298 A.

